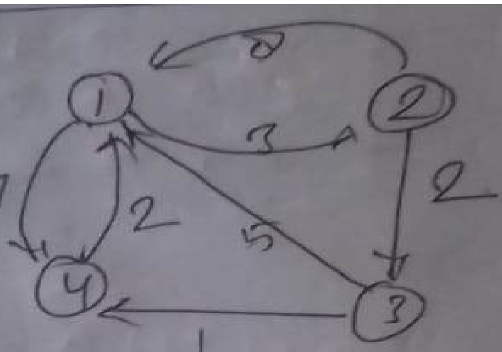
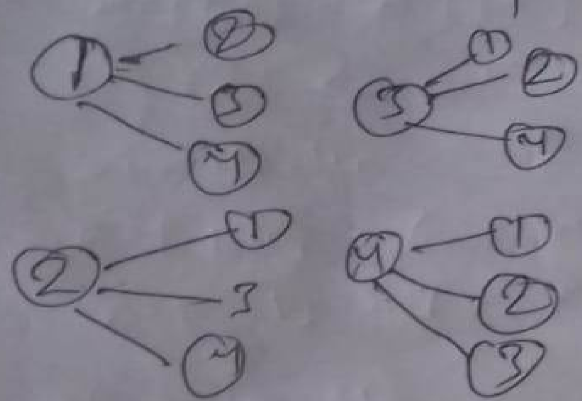
All pair shortest path problem:





$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 8 & \infty & 0 \end{array}$$

$A^1[2,3] \Rightarrow 2$

$$\underbrace{A^0(2,3)}_{2} < \underbrace{A^0(2,1) + A^0[1,3]}_{8+\infty}$$

$A^1[2,4] \Rightarrow 15$

$$\underbrace{A^0[2,4]}_{\infty} > \underbrace{A^0[2,1] + A^0[1;4]}_{8+7=15}$$

$A^1[3,2] \Rightarrow 8$

$A^0[3,2] \quad A^0[3,1] + A^0[1,2]$

$\infty > 5 + 3 = 8$

Similarly

$A^1[3,4] \Rightarrow 1$

$A^1[4,2] \Rightarrow 8$

$A^1[4,3] = \infty$

Now $A^2 =$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 8 & 7 & 0 \end{bmatrix} \end{array}$$

$A^2[1,3] \Rightarrow 5$

$A^1[1,3] \quad A^1[1,2] + A^1[2,3]$

$\infty > 3 + 2 = 5$

$A^2[1,4] \Rightarrow 7$

$A^1[1,4] \quad A^1[1,2] + A^1[2,4]$

$7 < 3 + 15$

Similarly

$A^2[3,1] \Rightarrow 5 \qquad A^2[4,1] \Rightarrow 2$

$A^2[3,4] \Rightarrow 1 \qquad A^2[4,3] = 7$

$$A^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 7 & 0 & 2 & 3 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$A^3[1,2] \Rightarrow 3$

$A^2[1,2] \quad A^2[1,3] + A^2[3,2]$

$3 < 5 + 8$

Similarly.

$A^3[1,4] \Rightarrow 6$, $A^3[2,4] = 3$, $A^3[4,2] \Rightarrow 5$

$A^3[2,1] \Rightarrow 7$, $A^3[4,1] \Rightarrow 2$,

Now in the similar way we can find

$$A^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 3 & 6 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

Formula is

$$A^k[i,j] = \min\left\{A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}[k,j]\right.$$

```
for(k=1; k<=n; k++)
{
 for(i=1; i<=n; i++)
  {
  for(j=1; j<=n; j++)
   {
   A[i,j]= min(A(i,j), A[i,k]+A[k,j]
   }
  }
}
```

$T(n) = \theta(n^3)$